Magnetic Penetration Depth in a Superconductor

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abstract

It is shown that the notion of the finite pairing interaction energy range T_d results in a linear temperature dependence of the London magnetic penetration depth $\Delta\lambda/\lambda(0) = (T/T_d)(2/\pi)\ln 2$ at low temperature, in the case of the s-wave pairing state.

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One of important parameters in a superconductor is the London magnetic penetration depth $\lambda(T)$ which reflects the condensed carrier density, superfluid density $\rho_s(T)/\rho_s(0) = [\lambda(0)/\lambda(T)]^2$. The temperature dependence of $\rho_s(T)$ plays an important role for understanding the nature of condensation. In the two fluid picture, $\rho_s(T)$ varies as $1-(T/T_c)^4$. But the BCS-$\rho_s(T)$ has an activation form at low temperature via the order parameter Δ, which indicates the excitation energy gap.

The measurements of $\lambda(T)$ at low temperature in high T_c superconductors (HTS) are compatible with neither the BCS result nor the two fluid picture. The data of $\lambda(T)$ in a single crystal YBCO [1] indicates a linear temperature dependence. On the other hand, the data in films [2] behaves the T-square dependence. The linear temperature dependence of $\lambda(T)$ is in fact taken as providing evidence that the order parameter has nodes, suggesting the d-wave pairing states [3]. The T-square behavior of $\lambda(T)$ is understood due to scatterings via impurities or defects [4]. Recently, the non-local effect in a pure d-wave superconductor [5] is suggested to have the T-square behavior of $\lambda(T)$ at very low temperature.

Contrary to the general belief, I show here that it is not necessary to have a node in the order parameter, to account for a linear temperature dependence of $\rho_s(T)$ at low temperature.

The fact is that to have a finite value of T_c, a finite pairing interaction energy range T_d is required within the pairing theory [6]. In other words, the order parameter $\Delta(k,\omega)$ may be given as

$$\Delta(k,\omega) = \left\{ \begin{array}{ll} \Delta & \text{for} \quad |\epsilon_k| < T_d \\ 0 & \text{for} \quad |\epsilon_k| > T_d \end{array} \right\} \tag{1}$$

for all frequencies ω. Here ϵ_k is the usual normal state excitation energy with the mo-

mentum k, measured with respect to the Fermi level. Here the units of $\hbar = c = k_B = 1$ are used.

By carrying out the ϵ_k-integration of the imaginary part of the usual Green's function [7] consistent with Eq.(1), the density of states $n(\omega) = N(\omega)/N(0)$ is obtained as [6]

$$
\begin{aligned}
n(\omega) &= q(\omega/T_d) + n_{\rm BCS}(\omega) r(\omega/T_d) &(2)\\
q(\omega/T_d) &= (2/\pi)\tan^{-1}(\omega/T_d) &(3)\\
r(\omega/T_d) &= (2/\pi)\tan^{-1}[n_{\rm BCS}(\omega)T_d/\omega] &(4)\\
n_{\rm BCS}(\omega) &= Re\{\omega/(\omega^2-\Delta^2)^{1/2}\}. &(5)
\end{aligned}
$$

The $q(\omega/T_d)$ is resulted from the ϵ_k-integration of the Green's function with $\Delta(k,\omega)=0$ for $|\epsilon_k| > T_d$. In the infinite T_d limit, $n(\omega) = n_{\rm BCS}(0)$ as expected.

In the spirit of the BCS, the London penetration depth $\Delta\lambda = \lambda(T) - \lambda(0)$ may be given as [3]

$$\Delta\lambda/\lambda(0) = \int_0^\infty d(\omega/T)\ n(\omega) f(\omega/T)[1-f(\omega/T)] \tag{6}$$

where $f(x) = 1/[1+\exp(x)]$ is the Fermi function. For the BCS density of states $n_{\rm BCS}(\omega)$, the well known result at low temperature follows

$$[\Delta\lambda/\lambda(0)]_{\rm BCS} = (2\pi\Delta/T)^{1/2}\exp[-\Delta/T]. \tag{7}$$

By inserting $q(\omega/T_d)$ of Eq.(3) into Eq.(6), at low temperature we get

$$[\Delta\lambda/\lambda(0)]_q = (T/T_d)(2/\pi)\ln 2, \tag{8}$$

similar to that resulted from the order parameter of the d-wave symmetry [3],

$$[\Delta\lambda/\lambda(0)]_d = (T/\Delta_0)\ln 2 \tag{9}$$

via $n_d(\omega) = \omega/\Delta_0$, where Δ_0 is the maximum value (anti-node) of the order parameter. The Eq.(8) is a reflection of the finite pairing interaction energy range T_d, that is, $q(\omega/T_d)$. In low T_c superconductor (LTS), the pairing interaction energy range $T_d \simeq T_c \exp(1/g)$ in the weak coupling g limit is large compared to T_c and makes the linear temperature dependence of $\lambda(T)$ hardly observable. On the other hand, in HTS, even though the exact nature of the pairing interaction is not known, T_d appears to be of the order of T_c. Thus, the linear temperature dependence of $\lambda(T)$ is observed [1]. In fact, the data of [1] yields $T_d \simeq 2T_c$ via Eq.(8).

In conclusion, the notion of the finite pairing interaction energy range results in a linear temperature dependence of superfluid density $\rho_s(T)$ at low temperature, which yields the T-power laws in the various properties in a superconductor [6]. A linear temperature dependence of $\lambda(T)$ does not imply nodes in the order parameter.

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